Factor trees

Composite numbers can be divided exactly (with no remainder), by other smaller or equal whole numbers called **factors**.

Composite numbers:	15	9	12	4	24
Factors:	1 , 3 , 5 , 15	1 , 3 , 9	1 , 2 , 3 , 4 , 6 , 12	1 , 2 , 4	1 , 2 , 3 , 4 , 6 , 8 , 12 , 24

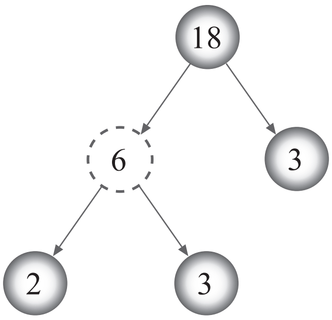
Prime numbers only have 1 and themself as factors.

Prime numbers:	2	3	17	11	31
Factors:	1 , 2	1 , 3	1 , 17	1 , 11	1 , 31

All composite numbers can be written as the product ($\times$) of **prime factors** (all the prime numbers that divide exactly into them). Let's see how.

'Express' is a another way of saying 'write' in Mathematics.

Express 18 as a product of its prime factors



Split 18 into two smaller factors

Solid circle around prime numbers to stop that branch

Split 6 into two smaller factors

Solid circle around prime numbers to stop that branch

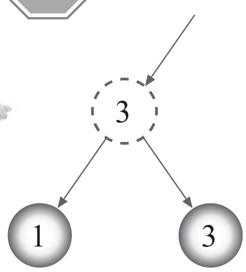
Once every branch has reached a prime number, multiply all the prime numbers together

$$\therefore 18 = 2 \times 3 \times 3$$
$$= 2 \times 3^2$$

Simplify answer

ALWAYS STOP at the prime number.

Don't ever do this

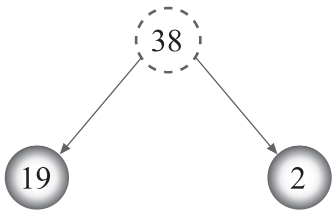


because 1 is NOT a prime number

Remember:
A prime number has two factors, itself and 1

Here are some more examples.

Express 38 as a product of its prime factors



Split 38 into two smaller factors

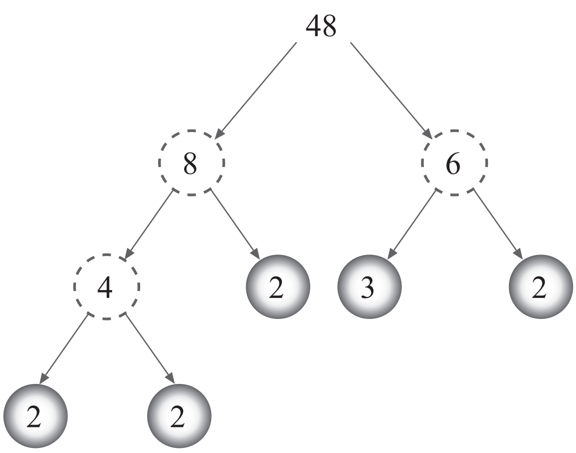
Solid circle around prime numbers to stop that branch

Once every branch has reached a prime number, multiply all the prime numbers together

$$\therefore 38 = 19 \times 2$$

There is often more than one way to create a factor tree for numbers with a lot of factors.

Express 48 as a product of its prime factors



Split 48 into two smaller factors

Split 6 and 8 into two smaller factors

Solid circle around prime numbers to stop that branch

Split 4 into two smaller factors

Solid circle around prime numbers to stop that branch

Once every branch has reached a prime number, multiply all the prime numbers together

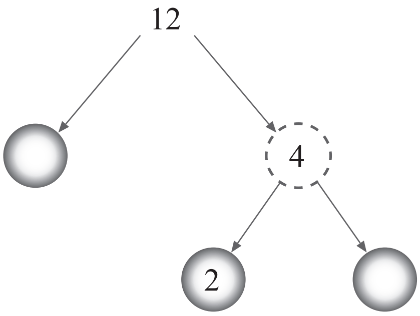
$$\therefore 48 = 2 \times 2 \times 2 \times 3 \times 2$$
$$= 2^4 \times 3$$

Simplify answer

Factor trees

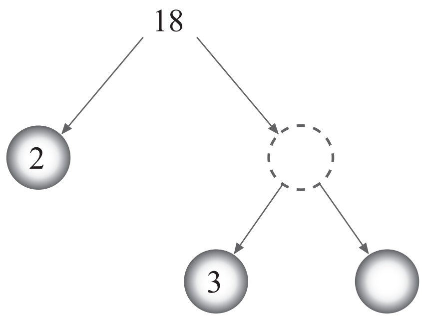
1 Fill in the missing values on the following factor trees and write the number as a product of its primes.

a



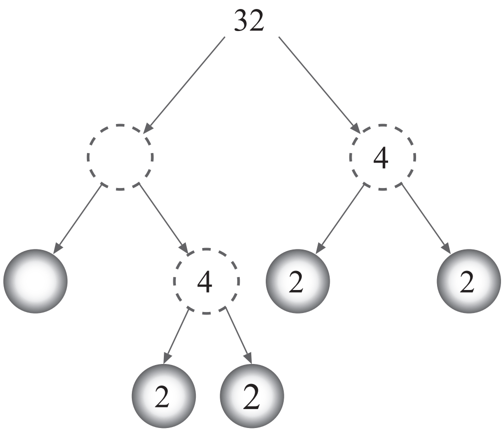
$\therefore 12 =$

b



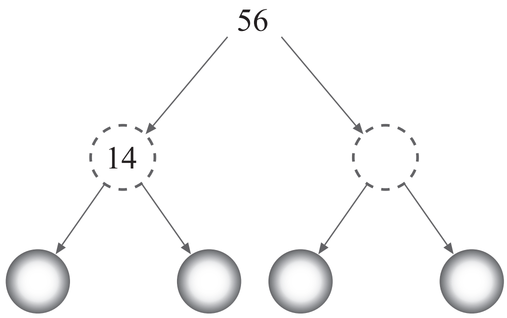
$\therefore 18 =$

c



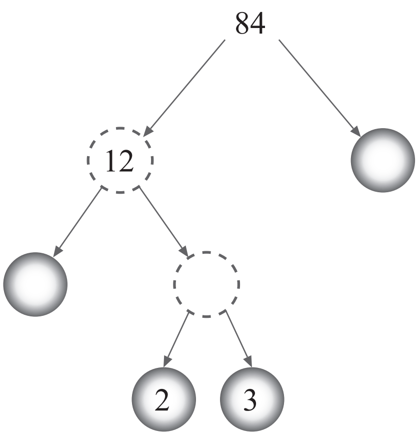
$\therefore 32 =$

d



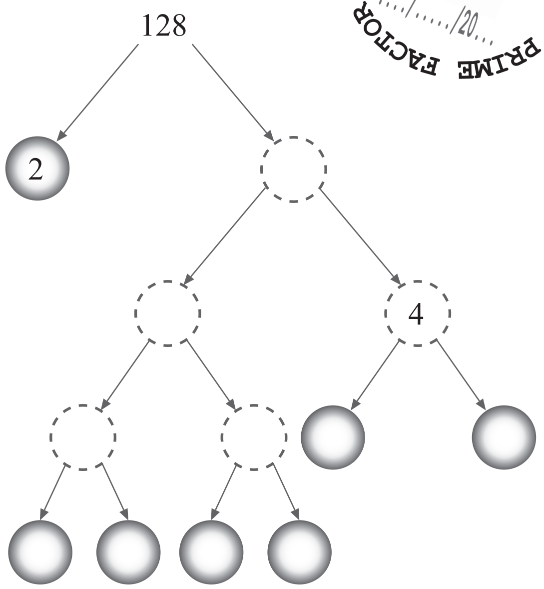
$\therefore 56 =$

e



$\therefore 84 =$

f



$\therefore 128 =$

Factor trees

2 Complete a factor tree for each number below and express them as a product of their prime factors.

a 8

$\therefore 8 =$

b 20

$\therefore 20 =$

c 24

$\therefore 24 =$

d 60

$\therefore 60 =$

e 96

$\therefore 96 =$

f 144

$\therefore 144 =$

Highest common factor (HCF)

The HCF is the largest number that divides exactly into two or more composite numbers.

Write all the factors of each number then circle the largest one which appears in both lists.

Find the highest common factor for these pairs of numbers

(i) 6 and 8

Factors of 6: 1 , 2 , 3 , 6 — List all the factors for each number

Factors of 8: 1 , 2 , 4 , 8 — Circle the largest number common to both lists

$\therefore$ The HCF for 6 and 8 is: **2**

(ii) 18 and 12

Factors of 18: 1 , 2 , 3 , 6 , 9 , 18 — List all the factors for each number

Factors of 12: 1 , 2 , 3 , 4 , 6 , 12 — Circle the largest number common to both lists

$\therefore$ The HCF for 18 and 12 is: **6**

We can use the list of prime factors for larger numbers to find the HCF.

Find the HCF for these pairs of larger numbers

(i) 72 and 96

Factors of 72: 2 , 2 , 2 , 3 , 3 — List all the **prime** factors for each number

Factors of 96: 2 , 2 , 2 , 2 , 2 , 3

$\therefore$ The HCF for 72 and 96 is: $2 \times 2 \times 2 \times 3 = \mathbf{24}$

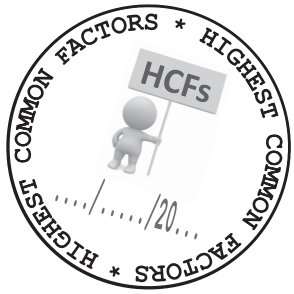
(ii) 528 and 624

Factors of 528: 2 , 2 , 2 , 2 , 3 , 11 — List all the **prime** factors for each number

Factors of 624: 2 , 2 , 2 , 2 , 3 , 13

$\therefore$ The HCF for 528 and 624 is: $2 \times 2 \times 2 \times 2 \times 3 = \mathbf{48}$

Highest common factor (HCF)



1. Find the highest common factor for these pairs of numbers.

a. 8 and 12

b. 6 and 15

c. 10 and 18

d. 18 and 24

e. 14 and 28

f. 16 and 36

2. Use the prime factors to find the HCF for these larger numbers.

a. 42 and 84

b. 92 and 72

c. 280 and 490

d. 256 and 640

Lowest common multiple (LCM)

The LCM is the smallest number that is common to the multiplication tables of two or more numbers.

Write down the multiples of the numbers and stop once you find the lowest common multiple.

Find the lowest common multiple for these pairs of numbers

(i) 2 and 5

Multiples of 2: $\underset{1\times 2}{2}$, $\overset{2\times 2}{4}$, $\underset{3\times 2}{6}$, $\overset{4\times 2}{8}$, $\underset{5\times 2}{10}$, $\overset{6\times 2}{12}$, $\underset{7\times 2}{14}$,... List some multiples of the first number

Multiples of 5: $\underset{1\times 5}{5}$, $\overset{2\times 5}{10}$,... List the multiples of the second number until there is a match

$\therefore$ The LCM for 2 and 5 is: $\mathbf{10}$

(ii) 6 and 8

Multiples of 6: 6 , 12 , 18 , 24 , 30 ,... List some multiples of the first number

Multiples of 8: 8 , 16 , 24 ,... List the multiples of the second number until there is a match

$\therefore$ The LCM for 6 and 8 is: $\mathbf{24}$

We can use the list of prime factors for larger numbers to find the LCM by looking at the differences.

Find the LCM for these pairs of larger numbers

(i) 30 and 100

Prime factors of 30: 2 , 3 , 5 List all the **prime** factors for both numbers

Prime factors of 100: 2 , 2 , 5 , 5 Circle **all** the **different** factors in the smaller number

$\therefore$ The LCM for 30 and 100 is: $100 \times \mathbf{3} = 300$ Multiply the larger number by the **different** factor

(ii) 24 and 388

Prime factors of 24: 2 , 2 , 2 , 3 List all the **prime** factors for both numbers

Prime factors of 388: 2 , 2 , 97 Circle **all** the **different** factors in the smaller number

$\therefore$ The LCM for 15 and 388 is: $388 \times \mathbf{2} \times \mathbf{3} = 2328$ Multiply the larger number by the **different** factors

Lowest common multiple (LCM)

1 Find the lowest common multiple for these pairs of numbers.

a 3 and 9

b 5 and 10

c 4 and 6

d 5 and 6

e 6 and 7

f 12 and 16

2 Use the prime factors to find the LCM for these larger numbers.

a 60 and 108

b 42 and 150

c 168 and 180

d 210 and 385

Types of numbers – prime and composite numbers

Eratosthenes (276 BC – 194 BC) was a Greek mathematician who developed a clever way to find prime numbers.

3 **Find all the prime numbers in the hundred grid below. (Do not shade the number itself as it is not a multiple.)**

a Cross out 1 since it is not prime.

b Shade all the multiples of 2.

c Shade all the multiples of 3.

d Shade all the multiples of 5.

e Shade all the multiples of 7.

f The remaining numbers are prime numbers, apart from 1 which is a special case. List them:

____

____

The Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

4 **Circle the prime numbers. Use the Sieve of Eratosthenes to help you.**

65	89	47	94	25	43
11	27	32	19	21	65
7	53	99	87	26	13

Types of numbers – mixed practice

1 **Work out what the secret numbers are. Assume all numbers are positive, unless stated otherwise.**

a I am the only even prime number. I am ____________.

b I am one of the two numbers that are neither prime nor composite. I am not zero.

I am ____________.

c I am a 2 digit number. I am less than 40. I am a prime number and my second digit is smaller than my first number. I am ____________.

d I am the negative number closest to positive numbers. I am ____________.

e I am the 5 digit negative number furthest from zero. I am ____________.

f I am the largest 5 digit number where no number is repeated. I am ____________.

g I am the largest 4 digit number that uses the 4 smallest prime numbers. I am ____________.

h I am a prime number. My digits add to total the smallest prime number. I am ____________.

2 **In these next questions, there is more than 1 possible answer.**

a Look at the number 1 000 855.

Write 5 numbers that are larger than this with the same number of digits.

____

Write 5 numbers that are smaller.

____

b Rounded to the nearest 100 km, my train trip was 3 000 km long. How long could it have been? How many answers to this question can you find?

Goldbach's conjecture — investigate

Getting ready

In the year 1742, a Prussian mathematician called Christian Goldbach looked at many sums and made a conjecture. He said that every even number over 4 is the sum of 2 prime numbers. (Actually he said over 2 but that was when 1 was considered a prime number. That is now so 1742.)

What to do

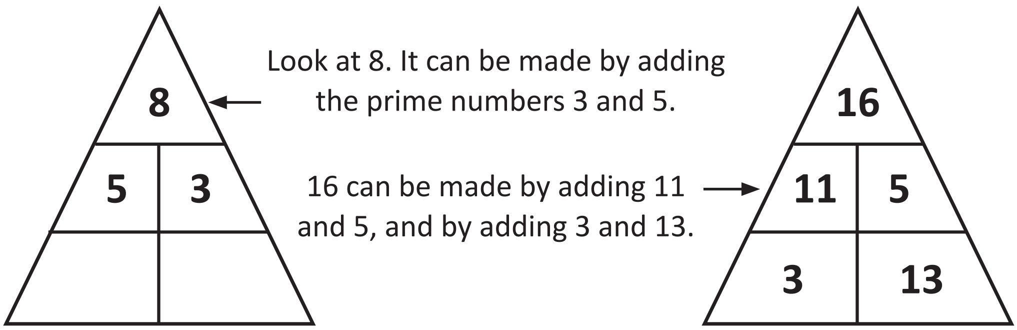
You have been asked by the Mathematics Institute to test this out.

How high can you go?

What will you need to help you solve this problem? You may want to use the table of prime numbers on page 18.

You can work by yourself or as part of a small group.

Here are a few to start you off.



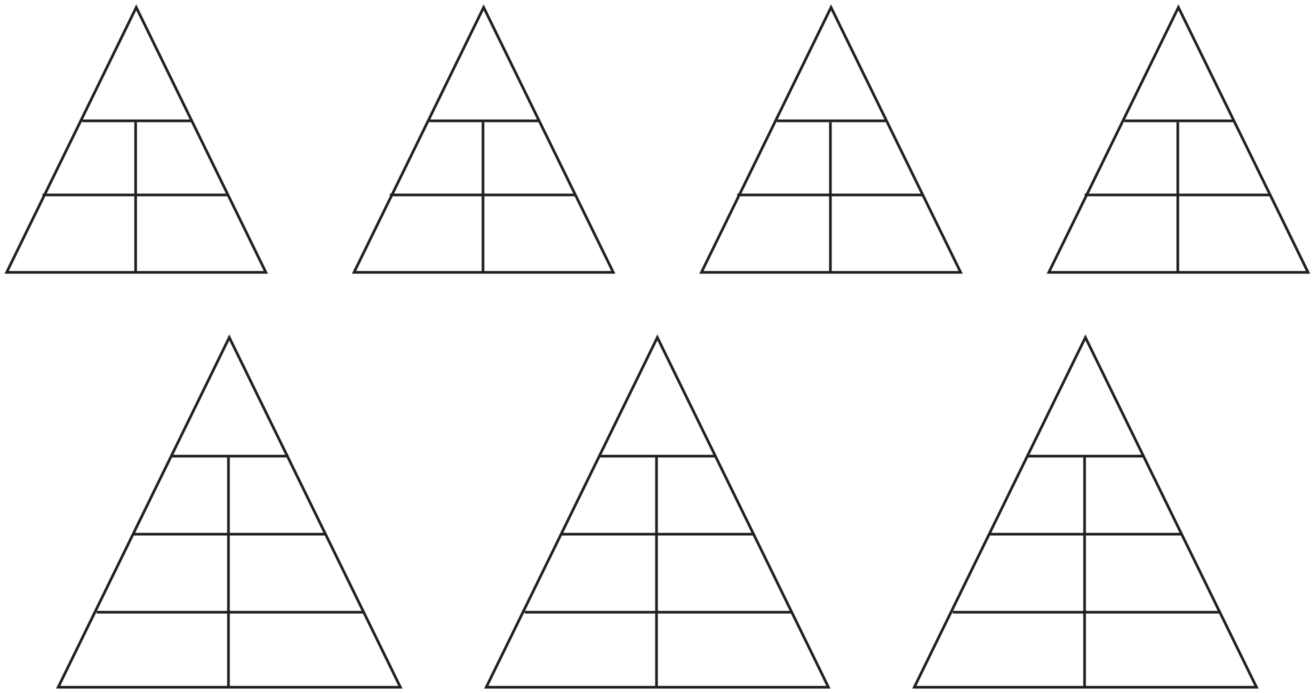
Use the triangles on page 18 to record your thinking. Or create your own.
You may need more!

What to do next

Which even number can be made the most ways? Discuss your answer with 2 friends. Do they agree?

Goldbach's theory has never been absolutely proven or disproven. The publishing group Faber and Faber offered a $1 000 000 prize to any one who could do so.
No one was able to claim the prize at the end of the competition time. Who knows, you could be the one to claim the glory (if not the prize). You could rename the conjecture. What would you call it?

Goldbach's conjecture — investigate



2	3	5	7	11	13
17	19	23	29	31	37
41	43	47	53	59	61
67	71	73	79	83	89
97	101	103	107	109	113
127	131	137	139	149	151
157	163	167	173	179	181
191	193	197	199	211	223
227	229	233	239	241	251
257	263	269	271	277	281
283	293	307	311	313	317
331	337	347	349	353	359
367	373	379	383	389	397
401	409	419	421	431	433
439	443	449	459	461	463
467	479	487	491	499	